Brownbag Lunch, 04.05.07
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Gibbard-Sattherwaite-Theorem

Gibbard, Alan. (1973): "Manipulation of Voting Schemes: A General Result." *Econometrica* 41(4): 587-602.

Sattherwaite, Mark (1975): "Strategy-proofness and Arrows Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions" *Journal of Economics Theory* 10: 187-217.

Non-technical Meaning

All non-dictatorial decision making procedures can potentially be manipulated through strategic voting.

Technical Formulation

If the decision set contains more than 2 elements and the social choice function (G) is non-manipulable, then G constitutes a dictatorship.

Assumptions of the Theorem

- n voters; $0 \leq n \geq \infty$
- number of alternatives m $\geqslant 3$
- voters may vote strategically

Explanation

The theorem starts from the premise that strategic voting distorts the aggregation of preferences and therefore leads to biased decisions that do not reflect the "true" preferences of the social entity.

Example: Suppose there are three alternatives x, y, z. If pitted against each other in a pairwise vote, majority rule would yield xPy, zPx, yPz. Hence, if x is first set up against y, then the collective decision at the end will be z. All voters with the preference order xPyPz have an incentive to vote strategically by voting for y instead of x in the first vote (i.e., they misrepresent their true preferences.). Then, y would defeat x and would go on to defeat z in the next vote. This way, these strategic actors could secure their second best outcome (y) instead of their least preferred one (z).

The question arises whether there is a decision rule that prevents actors from voting strategically. Gibbard and Sattherwaite show that the only rule that is capable of yielding a clear outcome and that cannot be manipulated through strategic voting is a dictatorial rule. The mathematical proof is related to Arrows impossibility theorem. It proceeds by showing that a strategy-proof social choice function that yields a transitive preference order and is independent of irrelevant alternatives, also satisfies Arrows weak Pareto criterion. As a result (according to Arrows theorem) there must be a dictator.