McKelvey’s Theorem


Non-technical Meaning:

In a situation of majority voting with three or more voters and at least two policy dimensions (i.e. two or more issues to be decided on), any policy outcome is theoretically possible, regardless of the initial policy status quo. As a result, through a finite number of pairwise voting steps, an agenda setter is able to move the outcome to any point he likes in the decision space.

Technical Formulation:

Given sincere voting, Euclidean preferences, an n-dimensional policy space and at least three voters, simple majority rule leads to an empty core and the cyclical top cycle set covers the whole policy space. As a result, for any two points x and y, there is a finite amendment agenda leading from x to y and back again to x.

Assumptions of the theorem:

- n voters, n ≥ 3
- voters have Euclidean preferences, i.e. u_i(x) = - |x - y_i|
- all voters vote sincerely
- n-dimensional policy-space, n ≥ 2
- voting by majority
- Plott’s (very special) condition of a radial symmetric preference distribution does not hold

Explanation:

Under these conditions there does not exist a single point (a policy package) in the policy space that cannot be beaten by another point (the core is empty). Therefore, under perfect information and with rational foresight, an agenda setter can devise a sequence of votes that will get him to any outcome he likes. In the example below, a clever agenda setter is able to move the outcome from point a (which lies within the Pareto set) in four voting steps to point e: 1 and 2 prefer b to a, but then 2 and 3 will be in favour to move from b to c; d is then, however, preferred by 1 and 3, and, finally, both 1 and 2 find it in their interest to move from d to e.
Scientific implication:

Any decision making model that only takes preferences and a majority voting rule as primitives is very likely not able to yield any predictions and is therefore underspecified.

Political implication:

The power of an agenda setter can be quite sizeable, leading in the extreme to something akin to a dictatorial rule. As a result, the power of the agenda-setter needs to be institutionally restricted, thus preventing him from rigging the outcome in his favour.