May’s Theorem: The only group decision function that satisfies decisiveness, anonymity, neutrality and positive responsiveness is simple majority rule.

Definitions: group decision function: \( D = f(D_1, D_2, \ldots, D_n) \), \( n \) is the number of individuals in the group. Each \( D_i \) takes on the value 1, 0 or \(-1\) and corresponds to the preferences of the individuals. Hence, 1 means that individual \( i \) strictly prefers \( x \) to \( y \), \(-1\) means that individual \( i \) strictly prefers \( y \) to \( x \) and 0 means that individual \( i \) is indifferent between \( x \) and \( y \). Thus, each \( D_i \) corresponds to a ballot an individual \( i \) casts between two alternatives. \( F(.) \) represents an aggregation rule that determines the winning issue.

Simple majority rule can thus be defined in the following way:

\[
\begin{align*}
\sum_{i=1}^{n} D_i > 0 & \rightarrow D = 1 \\
\sum_{i=1}^{n} D_i = 0 & \rightarrow D = 0 \\
\sum_{i=1}^{n} D_i < 0 & \rightarrow D = -1
\end{align*}
\]

In words, this definition means that \( x \) is preferred to \( y \) by the group, if the number of individuals strictly preferring \( x \) to \( y \) is higher than the number of individuals strictly preferring \( y \) to \( x \). Some authors prefer to call this aggregation rule plurality rule! Simple majority rule is then defined as: the group strictly prefers \( x \) to \( y \) iff the number of individuals strictly preferring \( x \) to \( y \) is higher than \( n/2 \).

- **Decisiveness:** For all preference profiles the group decision function is defined and single-valued.
- **Anonymity:** \( D \) is determined only by the values of \( D_i \), and is independent of how they are assigned. Any permutation of these ballots leaves \( D \) unchanged.
- **Neutrality:** If \( x \) defeats (ties) \( y \) for one set of individual preferences, and all individuals have the same ordinal rankings for \( z \) and \( w \) as for \( x \) and \( y \) then \( z \) defeats (ties) \( w \).
- **Positive responsiveness:** If \( D \) equals 0 or 1, and one individual changes his preferences from \(-1\) to 0 or 1, or from 0 to 1, and all other individual preference relations remain unchanged, then \( D = 1 \). Some authors prefer to call positive responsiveness monotonicity!

References