Gibbard's Theorem

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Gibbard's Theorem

Gibbard's Theorem:
If an aggregation rule is quasi-transitive, weakly Paretian and independent of irrelevant alternatives, then it is oligarchic.

Definition: Aggregation

Aggregation rule: An aggregation rule is a mapping

$$f : \mathcal{R}^n \to \mathcal{B},$$

where $\mathcal{R}^n$ is the set of all possible preference profiles and $\mathcal{B}$ is the set of all possible complete orderings over the set of alternatives.

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Nontechnically a preference aggregation rule is simply a decision-mechanism which takes the preferences of the individuals in a committee and aggregates those preferences in a complete collective preference ordering over the alternatives.

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Quasi-transitivity and completeness of the collective preference ordering jointly guarantee that there is an optimal alternative.

Definition: Weak Pareto Optimality

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Nontechnically weak Pareto optimality means that if all individuals agree that x is strictly better than y, then under the preference aggregation rule f, x should collectively be strictly preferred to y.

Definition: Independence of irrelevant alternatives

An Aggregation rule is *independent of irrelevant alternatives*, if for all $p, p' \in \mathcal{R}^n$ such that xR_iy iff xR'_iy for all $i \in N$, it follows that xRy iff xRy for all $x, y \in X$.

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Nontechnically independence of irrelevant alternatives means that the collective preference relation regarding two alternatives x and y only depends on the individual preferences with respect to x and y.

Definition: Oligarchy

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An individual i has a *veto* for x against y if for all $p \in \mathcal{R}^n$, xP_iy implies that yPx does not hold. An individual i has a veto, if i has a veto for x against y, for all $x, y \in X$.

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A set L is *decisive* if for all $p \in \mathcal{R}^n$, $[xP_iy$, for all $i \in N]$ implies xPy.

Bibliography

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